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**COMMENTS ON POWER SPECTRA OF
DISCRETE STOCHASTIC TIME SERIES***

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**COMMENTS ON POWER SPECTRA OF
DISCRETE STOCHASTIC TIME SERIES***

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ABSTRACT

We show that the slope of slightly flatter than -2 seen in the power spectra of stochastic series is a consequence of finite discrete systems observed with limited temporal correlation.

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In lattice models, one describes nature at the microscopic level in terms of a discrete space time, and then obtains a macroscopic level description by coarse graining.¹ In this note, we wish to point out that coarse graining itself implies the power law spectra often measured for stochastic systems. Furthermore, the resultant exponent of slightly flatter than -2 is a signature of a discrete rather than continuous system.

To fix our ideas, consider a stochastic time series $f_j = f(t_j)$, $j=1$ to N , with discrete Fourier transform $g(\omega)$:

$$g(\omega) = \sum_j f_j \exp(i\omega t_j) \quad (1)$$

Here $|g(\omega)|^2$ is the power spectrum; its ensemble average is independent of frequency ω for a white noise source. We attempt to measure this process on the macroscopic level with a physical device of limited temporal resolution. Suppose the impulse response of the device is $W(t)$, then the measured signal will be $F(t) = f(t)*W(t)$ with power spectrum $|F(\omega)|^2 = |g(\omega)|^2 |V(\omega)|^2$. Here, the star denotes convolution and $V(\omega)$ is the Fourier transform of the impulse response $W(t)$.

The simplest possible case is that of a running average over a time interval T ie: $W(t) = 1$ for $0 < t < T$ and 0 otherwise. In this case, the measured power spectrum is proportional to

$$|V(\omega)|^2 = (\sin \omega T/2 / \sin \omega \Delta t/2)^2 \quad (2)$$

This function is plotted in Fig. 1 for $N=2048$ and a window of $T=200 \Delta t$. It is seen that the spectrum displays an apparent power law behavior in the high frequency range with exponent slightly flatter than -2. The apparent leveling at the

highest ω is due to the finite spectral width and periodicity of the finite Fourier transform. The approximate slope of -1.7 is nearly independent of the size of the window as long as the window T is small relative to N . Physically, we can think of T as a correlation time. Then statistically relevant results require $T \ll N\Delta t$. The continuum limit should be approached when $\Delta t \ll T \ll N\Delta t$. For a smaller T , the effect of discreteness will be manifested.

This result can be understood by taking the logarithmic derivative $d \ln |V(\omega)|^2 / d \ln \omega$. The slowly varying envelope seen in Fig. 1 is due to the denomination of Eq. 2. The resulting slope varies from -2 to about -1.56 as $\omega \Delta t$ varies over the physically meaningful range; the average value is -1.86 . This result is very suggestive in light of experimentally determined power spectra of turbulent systems on the one hand, and fluctuations near a critical point on the other (the critical exponent for the correlation function is $-2 + \eta$ where $\eta \approx .14$ experimentally).

As a demonstration of these ideas, we constructed a finite stochastic time series of $N = 2048$ values with a random number generator. The power spectrum of the series was found to be flat as expected. The "measured" time series was generated by using a running average of width $200 \Delta t$. The trend can be made more evident by performing a running average in frequency as well; this has been done in Fig. 2 over a frequency width of $20 \Delta \omega$. The slope of the spectrum is consistent with -1.7 .

We conclude that the slope of -2 is characteristic of a discrete process with limited time correlation, in distinction to the slope of -2 characteristic of a continuum.² Furthermore, the power law spectrum is a natural consequence of finite spatial or temporal resolution measurements of finite time series. The imposition of correlation through finite resolution observation is enough to lead to power law spectra.

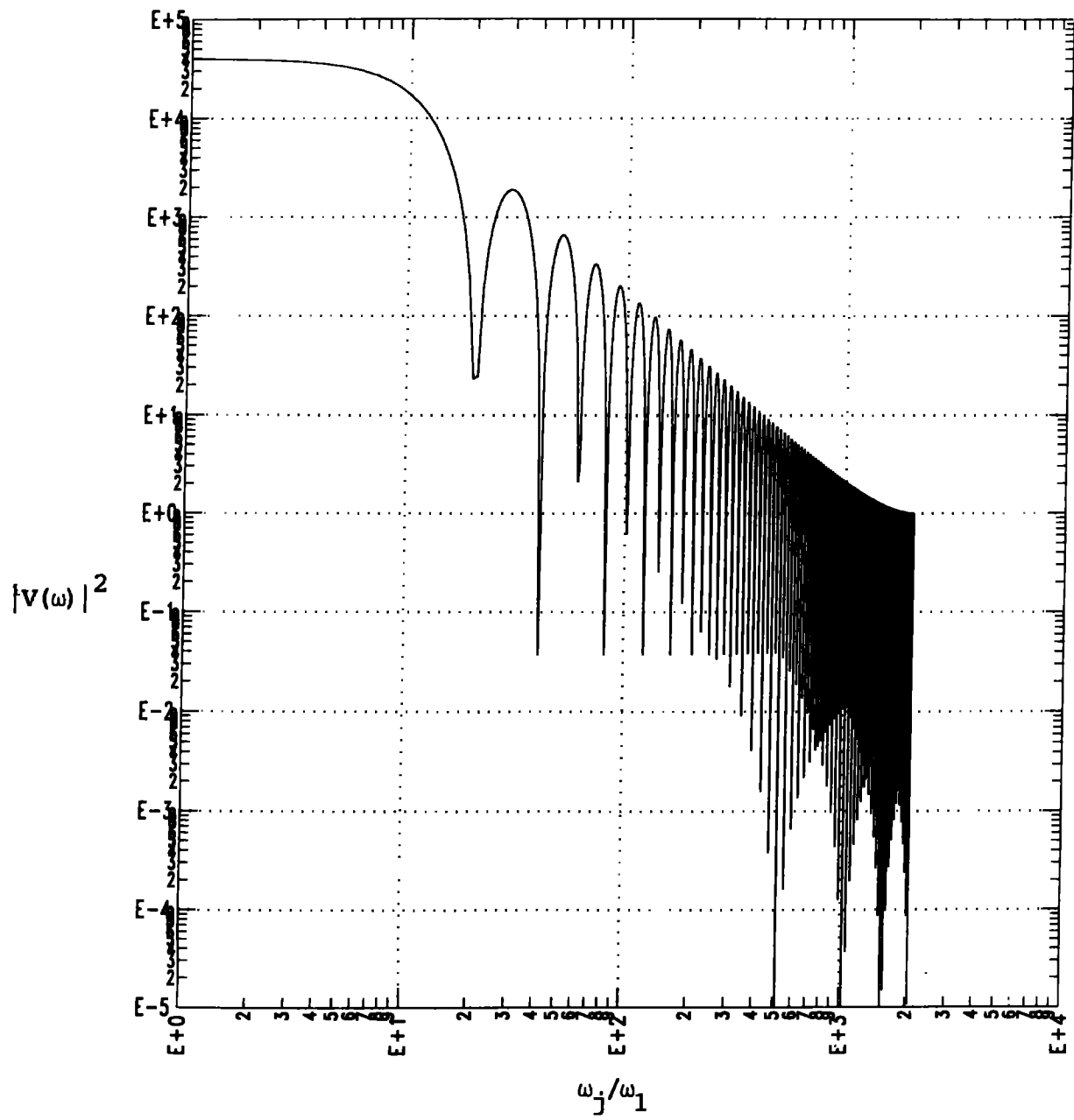


Fig.1

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2. For the continuum case, Eq. 2 becomes $|V(\omega)|^2 = (\sin \omega T/2)^2/(\omega/2)^2$.

FIGURE CAPTIONS

Fig. 1 Plot of $|V(\omega)|^2$ given by Eq. 2 for a time series with $N = 4096$ and $T = 200 \Delta t$.

Fig. 2 The power spectrum of a time series of $N = 4096$ random numbers with $T = 200 \Delta t$ and a averaging frequency window of $20 \Delta \omega$.

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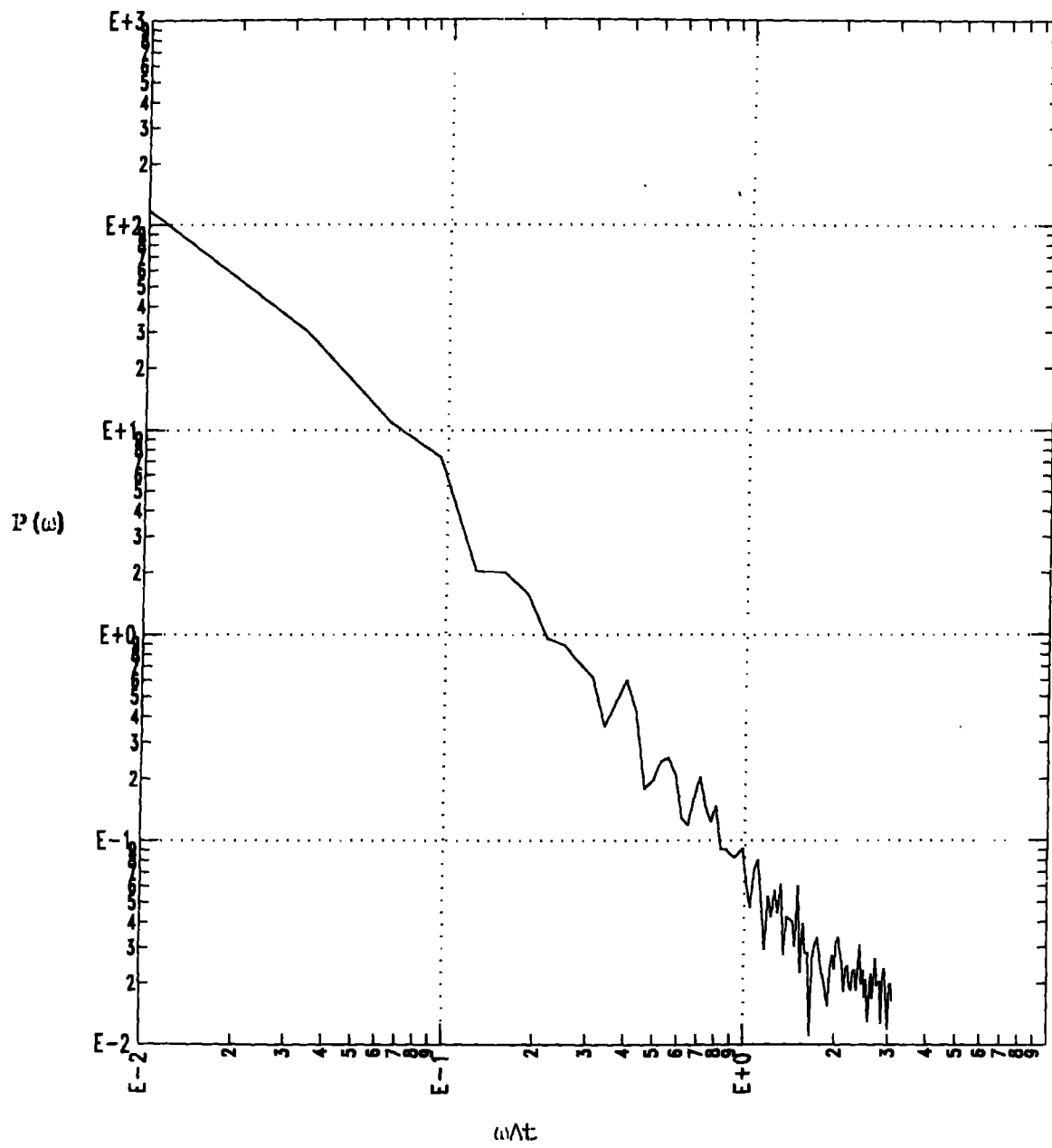


Fig. 2

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